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Effect of viscous dissipation on fully developed laminar mixed convection in a vertical double-passage channel

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Abstract

Fully developed laminar mixed convection in a vertical double-passage channel has been investigated numerically by taking into account the effect of viscous dissipation. Uniform wall temperatures with asymmetric and symmetric heating have been considered. An analytical solution has been developed for the case when viscous dissipation is neglected. The results showed that the increase in Brinkman number decreases the Nusselt number on the hot wall and increases that on the cold wall specially when the baffle becomes near the hot wall or the cold wall, respectively.

Keywords: Laminar flow; Mixed convection; Viscous dissipation; Channel flow; Brinkman number

1. Introduction

Enhancement of the heat transfer in a vertical channel is a major aim because of its practical importance in many engineering systems. The convective heat transfer in a vertical channel could be enhanced by using special inserts, which can be specially designed to increase the included angle between the velocity vector and the temperature gradient vector rather than to promote turbulence. This increases the rate of heat transfer without a considerable drop in the pressure [1]. A plane baffle may be used as an insert to enhance the rate of heat transfer in the channel. To avoid a considerable increase in the transverse thermal resistance into the channel, a thin and perfectly conductive baffle is used.

In a previous work [2], the effect of such baffle on the laminar fully developed combined convection in a vertical channel with different uniform wall temperatures has been studied analytically. Results showed that the presence of the baffle may lead to a significant enhancement of the heat transfer in the channel according to the baffle position and the value of Gr/Re. Higher values of Nu_h can be obtained when the baffle becomes as near the hot wall as possible, but at this position lower values of Nu_c are observed at the cool wall. To obtain higher rates of heat transfer on both walls of the channel, two baffles may be used instead of one baffle [3]. The numerical investigation of the developing laminar convection in the channel emphasized these results [4].

The studies available in the literature concerning the laminar convection in a vertical channel, except the work of Barletta [5,6] and Zanchini [7], neglect the effect of viscous dissipation. Barletta and Zanchini concluded that the effect of viscous dissipation could be important especially in the case of upward flow. Although viscous dissipation is usually neglected in low-speed and low-viscosity flows through conventionally sized channels of short lengths, it may become important when the length-to-width ratio is large [8]. For double-passage channels, the length-to-width ratio becomes large as the baffle becomes near the wall. So, viscous dissipation may become important.

The aim of the present work is to study the effect of viscous dissipation on fully developed laminar mixed convection in a vertical double-passage channel. The problem will be analyzed numerically, while an analytical solution has been developed for the case when viscous dissipation is neglected.

2. Analysis

The channel shown in Fig. 1 is divided into two passages by means of a perfectly conductive and thin baffle. Consider-

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Nomenclature

b	channel width	m
b^*	width of passage 1	m
Br	Brinkman number $= \mu u_r^2/k(T_h - T_c)$	
c_p	specific heat at constant pressure	$J\cdot kg^{-1}\cdot K^{-1}$
g	gravitational acceleration	$m\cdot s^{-2}$
Gr	Grashof number $= g\beta(T_h - T_c)b^3/\nu^2$	
h	heat transfer coefficient	$W\cdot m^{-2}\cdot K^{-1}$
k	thermal conductivity	$W\cdot m^{-1}\cdot K^{-1}$
Nu	Nusselt number $= hb/k$	
Nu	Nusselt number with $Br = 0$	
p	pressure	Pa
Re	Reynolds number $= u_r b/\nu$	
T	temperature	K
u	axial velocity	$m\cdot s^{-1}$
x	axial coordinate	m
y	transverse coordinate	m
Y^*	$= b^*/b$	

Greek symbols

β	volumetric coefficient of thermal expansion	K^{-1}
γ	pressure gradient	
θ	dimensionless temperature	
μ	dynamic viscosity	Pa·s
ν	kinematic viscosity	$m^2\cdot s^{-1}$
ρ	density	$kg\cdot m^{-3}$

Subscripts

b	bulk
c	cold wall plate
h	hot wall plate
r	reference
1	value in stream 1
2	value in stream 2

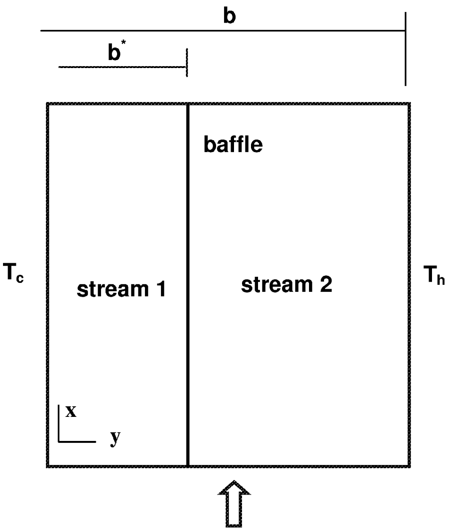

Fig. 1. Geometry and boundary conditions.

ation is given to a laminar, two-dimensional, incompressible, steady flow in the channel. The fluid enters the channel with a uniform upward vertical velocity and constant temperature. The channel walls are subjected to different constant temperatures, which are higher than that at the entrance. The fluid properties are assumed to be constant except for the buoyancy term of the momentum equation.

For fully developed flow, it is assumed that the transverse velocity and temperature gradient in the axial direction are zero. By taking into account the effect of viscous dissipation, the governing equations are

$$\nu \frac{d^2 u_i}{dy^2} = -g\beta(T_i - T_r) + \frac{1}{\rho_r}\frac{dp_i}{dx} \tag{1}$$

$$k\frac{d^2 T_i}{dy^2} + \mu\left(\frac{du_i}{dy}\right)^2 = 0 \tag{2}$$

The subscript 'i' denotes stream 1 or stream 2.

The boundary conditions are

$$y = 0: \quad u_1 = 0,\ T_1 = T_c \tag{3a}$$

$$y = b^*: \quad u_1 = u_2 = 0,\ T_1 = T_2 \tag{3b}$$

$$y = b: \quad u_2 = 0,\ T_2 = T_h \tag{3c}$$

The momentum balance equation (Eq. (1)) has been written according to the Boussinesq approximation by invoking a linearization of the equation of state, $\rho(T)$ around the reference temperature T_r. Recently, Barletta and Zanchini [9] have recommended the choice of the mean fluid temperature as the reference temperature in the fully developed region. In the present work, the mean temperature of the walls temperatures is chosen as the reference temperature, i.e.,

$$T_r = \frac{T_c + T_h}{2} \tag{4}$$

The governing equations can be expressed in the following dimensionless forms

$$\frac{d^2 U_i}{dY^2} = -\frac{Gr}{Re}\theta_i + \frac{dP_i}{dX} \tag{5}$$

$$\frac{d^2\theta_i}{dY^2} + Br\left(\frac{dU_i}{dY}\right)^2 = 0 \tag{6}$$

where

$$X = \frac{x}{bRe}, \qquad Y = \frac{y}{b}$$

$$U = \frac{u}{u_r}, \qquad \theta = \frac{T - T_r}{T_h - T_c}$$

$$P = \frac{p}{\rho_r u_r^2}, \qquad Re = \frac{u_r b}{\nu}$$

$$Gr = \frac{g\beta(T_h - T_c)b^3}{\nu^2}, \qquad Br = \frac{\mu u_r^2}{k(T_h - T_c)}$$

where the reference velocity u_r is defined as

$$u_r = \frac{1}{b}\int_0^b u\,dy \tag{7}$$

The pressure gradient in Eq. (5) is assumed to be constant, i.e.,

$$\frac{dP_i}{dX} = \gamma_i \tag{8}$$

The dimensionless boundary conditions are

$$Y = 0: \quad U_1 = 0,\ \theta_1 = -\frac{1}{2} \tag{9a}$$

$$Y = Y^*: \quad U_1 = U_2 = 0,\ \theta_1 = \theta_2 \tag{9b}$$

$$Y = 1: \quad U_2 = 0,\ \theta_2 = \frac{1}{2} \tag{9c}$$

For symmetric heating, i.e., $T_c = T_h$, the temperature difference $(T_h - T_c)$ in the definition of θ, Gr and Br is replaced by $\nu^2/c_p b^2$. Thus, the boundary conditions on θ are

$$\theta_1 \mid_{Y=0} = \theta_2 \mid_{Y=1} = 0 \tag{10}$$

Conservation of mass considered at any cross section of the channel passages gives

$$\int_0^{Y^*} U_1\,dY = Y^* \tag{11}$$

and

$$\int_{Y^*}^1 U_2\,dY = 1 - Y^* \tag{12}$$

The dimensionless bulk temperatures in streams 1 and 2 are defined as

$$\theta_{b1} = \int_0^{Y^*} U_1\theta_1\,dY \Big/ \int_0^{Y^*} U_1\,dY = \frac{1}{Y^*}\int_0^{Y^*} U_1\theta_1\,dY \tag{13}$$

$$\theta_{b2} = \int_{Y^*}^1 U_2\theta_2\,dY \Big/ \int_{Y^*}^1 U_2\,dY = \frac{1}{1 - Y^*}\int_{Y^*}^1 U_2\theta_2\,dY \tag{14}$$

The customary definitions of the Nusselt numbers, based on the bulk temperatures, are

$$Nu_c = \frac{2}{1 + 2\theta_{b1}}\frac{d\theta_1}{dY}\bigg|_{Y=0} \tag{15}$$

$$Nu_h = \frac{2}{1 - 2\theta_{b2}}\frac{d\theta_2}{dY}\bigg|_{Y=1} \tag{16}$$

Eqs. (15) and (16) show that Nu_c cannot be defined at $\theta_{b1} = -1/2$ and Nu_h cannot be defined at $\theta_{b2} = 1/2$. Therefore, the following definitions can be used [5,10]

$$Nu_c = \frac{b}{T_h - T_c}\frac{dT_1}{dy}\bigg|_{y=0} = \frac{d\theta_1}{dY}\bigg|_{Y=0} \tag{17}$$

$$Nu_h = \frac{b}{T_h - T_c}\frac{dT_2}{dy}\bigg|_{y=b} = \frac{d\theta_2}{dY}\bigg|_{Y=1} \tag{18}$$

For symmetric heating, the temperature difference $(T_h - T_c)$ in the definition of Nu_c and Nu_h is also replaced by $\nu^2/c_p b^2$.

Case of no viscous dissipation. If the viscous dissipation is negligible, i.e., $Br = 0$, the governing equations (5) and (6) can be solved analytically. The solution to these equations gives

$$\theta_i = Y - \frac{1}{2} \tag{19}$$

$$U_1 = \frac{1}{6}\frac{Gr}{Re}(Y^* - Y)\left[(Y^* + Y)Y - \frac{3}{2}Y\right] - \frac{\gamma_1}{2}(Y^* - Y)Y \tag{20}$$

$$U_2 = \frac{1}{6}\frac{Gr}{Re}\left[-Y^3 + \frac{3}{2}Y^2 - \frac{1}{2}(1 + Y^* - 2Y^{*2})Y + \frac{1}{2}(1 - 2Y^*)Y^*\right] + \frac{\gamma_2}{2}\left[Y^2 - (1 + Y^*)Y + Y^*\right] \tag{21}$$

Substitution from Eqs. (20) and (21) into Eqs. (11) and (12) gives the constants γ_1 and γ_2, i.e.,

$$\gamma_1 = -\frac{1}{2}\frac{Gr}{Re}(1 - Y^*) - \frac{12}{Y^{*2}} \tag{22}$$

$$\gamma_2 = \frac{1}{2}\frac{Gr}{Re}Y^* - \frac{12}{(1 - Y^*)^2} \tag{23}$$

Substitution from Eqs. (22) and (23) into Eqs. (20) and (21) leads to the following velocity profiles:

$$U_1 = \frac{1}{12}\frac{Gr}{Re}\left[-2Y^3 + 3Y^*Y^2 - Y^{*2}Y\right] + \frac{6Y}{Y^{*2}}(Y^* - Y) \tag{24}$$

$$U_2 = \frac{1}{12}\frac{Gr}{Re}\left[-2Y^3 + 3(1 + Y^*)Y^2 - (1 + 4Y^* + Y^{*2})Y + (1 + Y^*)Y^*\right] - \frac{6}{(1 - Y^*)^2}\left[Y^2 - (1 + Y^*)Y + Y^*\right] \tag{25}$$

Substituting Eqs. (19) and (24) into Eq. (13) yields

$$\theta_{b1} = \frac{1}{720}\frac{Gr}{Re}Y^{*4} - \frac{1}{2}(1 - Y^*) \tag{26}$$

Similarly, the dimensionless bulk temperature in stream 2 is

$$\theta_{b2} = \frac{1}{720}\frac{Gr}{Re}(1 - Y^*)^4 + \frac{1}{2}Y^* \tag{27}$$

It may be noted that the solution for the single-passage channel, neglecting the effect of viscous dissipation, can be obtained at $Y^* = 1$ in stream 1 or at $Y^* = 0$ in stream 2.

At sufficiently large Gr/Re, negative velocity and hence a flow reversal may be occurred adjacent to the cooler wall. The condition for occurring such flow in passage 1 is

$$\left.\frac{dU_1}{dY}\right|_{Y=0} < 0 \tag{28}$$

From Eq. (24) the above condition is formed as

$$\frac{Gr}{Re} > \frac{72}{Y^{*3}} \tag{29}$$

The condition for occurring a flow reversal in passage 2 is

$$\left.\frac{dU_2}{dY}\right|_{Y=Y^*} < 0 \tag{30}$$

From Eq. (25) the above condition is formed as

$$\frac{Gr}{Re} > \frac{72}{(1-Y^*)^3} \tag{31}$$

By putting $Y^* = 1$ in Eq. (29) or $Y^* = 0$ in Eq. (31), one can obtain a similar condition to that deduced by Aung and Worku [11] for a single-passage channel, taking into consideration the difference in the choice of the reference temperature. Aung and Worku, and the author too [2,3], have chosen the temperature at the channel entrance, instead of the mean fluid temperature, as the reference temperature.

3. Method for numerical solution

Investigation of the governing equations formulated above shows that the energy equation and the momentum equation are coupled and non-linear. Therefore, they can be solved by use of an iterative scheme. The equations are first approximated by finite difference equations. Then, the following procedure is used:

(1) Guess a temperature field at the interior points inside the whole channel.
(2) Solve the momentum equation, using Eqs. (11) and (12), to obtain the velocity field and the pressure gradient for the two streams separately.
(3) Use the values of the velocity of the previous step to solve the energy equation.
(4) Return to step 2 and repeat until convergence.

A uniform grid in the Y-direction is used with 41 grid points for each passage. Thus, smaller grid size in the narrower passage will be obtained. The solution procedure and grid sizes were validated by comparison with the analytical solution when viscous dissipation is neglected, for which $Nu_c = Nu_h = 1$. The results were in excellent agreement as can be seen from the following examples:

$Y^* = 0.2$: $Nu_c = 0.9999931$, $Nu_h = 1.000001$

$Y^* = 0.5$: $Nu_c = 0.9999979$, $Nu_h = 0.9999955$

$Y^* = 0.8$: $Nu_c = 1.000003$, $Nu_h = 0.9999901$

Furthermore, the calculations were carried out for the flow without buoyancy forces, i.e., $Gr/Re = 0$. For this case the analytical solution to the momentum equation gives

$$\gamma_1 = -\frac{12}{Y^{*2}} \quad \text{and} \quad \gamma_2 = -\frac{12}{(1-Y^*)^2} \tag{32}$$

The numerical calculations give very close values to those calculated using Eq. (32). For example, at $Y^* = 0.5$ Eq. (32) gives ($\gamma_1 = \gamma_2 = -48$) while the numerical calculations give ($\gamma_1 = \gamma_2 = -47.99993$). This confirms the accuracy and convergence of the numerical calculations.

4. Results and discussion

The governing equations with the given boundary conditions show that the numerical calculations depend on three parameters: Gr/Re, Br and Y^*. When the parameter Gr/Re exceeds a threshold value, flow reversal may occur causing downflow along the cooler wall. Examination of the flow reversal phenomenon is beyond the scope of this study. Therefore, the solution procedure was immediately stopped when a negative velocity has been attained. For more discussions of flow reversal phenomenon see Aung and Worku [11,12].

In the following sections the effect of Brinkman number on the Nusselt numbers on the channel's walls will be presented at different positions of the baffle with asymmetric and symmetric heating. First, the velocity and temperature profiles will be presented because of their usefulness in clarification of the heat transfer mechanism in the channel.

4.1. Asymmetric heating

The effect of Br on the velocity profiles for asymmetric heating is shown in Fig. 2 at different positions of the baffle. The figure reveals that the effect of Br appears clearly in the wider passage, while in the narrower passage its effect is not noticeable. When the baffle is in the middle of the channel slight effects of viscous dissipation can be noticed in both passages. The maximum velocity point moves towards the baffle as Br increases.

The energy equation shows that the temperature profile depends on the velocity profile for $Br \neq 0$. Since the velocity profiles depend on the baffle position, it is expected that the temperature profiles depend on the baffle position. Also, nonlinear temperature profiles are expected. Fig. 3 shows the temperature profiles at different positions of the baffle. The figure reveals that the temperature increases significantly in the whole channel due to the viscous heating. Also, the nonlinearity of temperature profiles increases as Br increases.

Indeed the increase in Br increases the temperature and hence the velocity increases due to the increase in thermal buoyancy force, but at the same time the increase in Br increases the pressure gradient, which decreases the velocity. For the narrower passage, in which the length-to-width

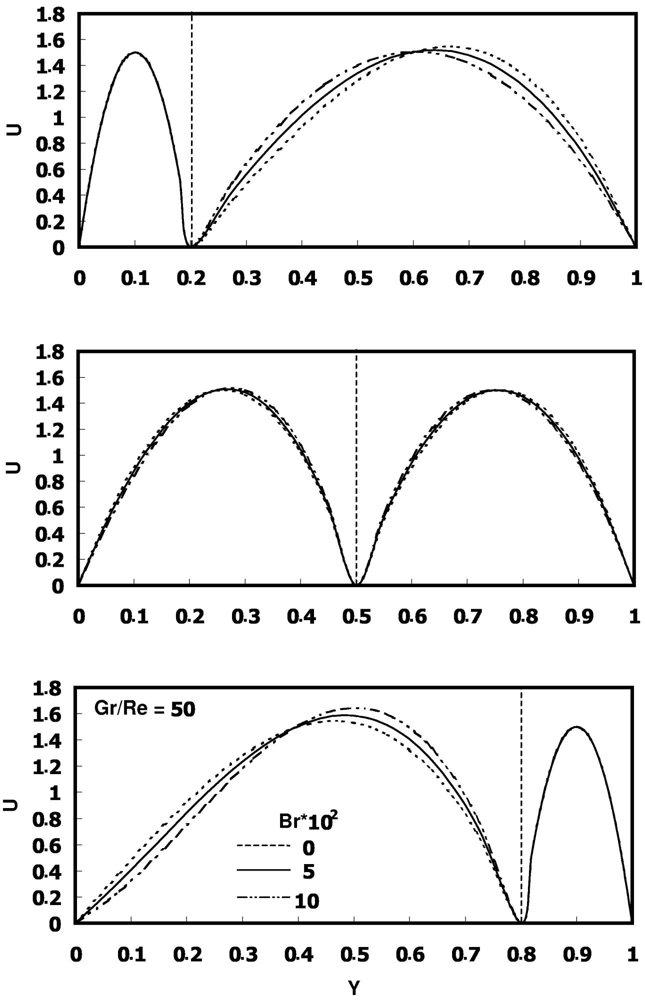

Fig. 2. Velocity profiles at different positions of the baffle (asymmetric heating).

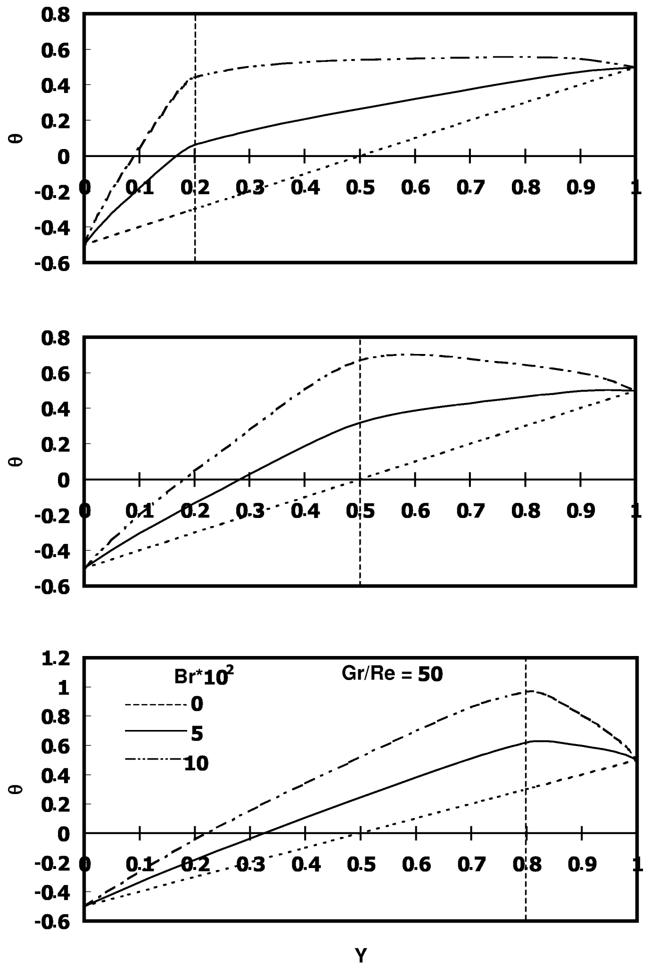

Fig. 3. Temperature profiles at different positions of the baffle (asymmetric heating).

ratio is large, the increase in the pressure gradient is high enough to resist the increase in thermal buoyancy force. So, Br has insignificant effect on the velocity profile. For the wider passage, the high increase in the temperature near the baffle makes the thermal buoyancy force the dominant force. Hence, the velocity increases with Br near the baffle. Consequently, the velocity decreases with Br near the wall, since the mass flow is constant at any cross section of the channel passages.

As indicated before, a flow reversal may be occurred adjacent to the cooler wall at sufficiently large Gr/Re. When passage 1 is the wider one, an increase in Br leads to a decrease in the velocity near the wall. This means that viscous dissipation enhances the chance of occurrence of flow reversal in this passage. Conversely, when passage 2 is the wider passage an increase in Br leads to an increase in the velocity near the baffle, i.e., the cooler side. This means that viscous dissipation lowers the chance of occurrence of flow reversal in passage 2.

To emphasize the above result, the critical values of Gr/Re when $Br = 0$ were calculated from Eqs. (29) and (31) at different positions of the baffle. The numerical calculations were carried out at $Br > 0$, using the calculated values of Gr/Re. A flow reversal has occurred in passage 1 at $Y^* \geqslant 0.5$, while at $Y^* \leqslant 0.5$ no flow reversal has occurred in passage 2.

Fig. 4 illustrates the variations of the ratio Nu_c/Nu_c with Br at different positions of the baffle. From the figure it is evident that the Nusselt number on the cool wall increases linearly with the increase in Br. This effect increases as the baffle becomes near the cool wall that is the length-to-width ratio of passage 1 becomes large. This can be explained with the aid of Fig. 3, which reveals that the increase in Br increases the temperature gradient and hence Nu_c increases at the cool wall regardless of the position of the baffle.

The variations of Nu_h/Nu_h with Br are shown in Fig. 5. In contrast to the cool wall, the Nusselt number on the hot wall decreases linearly with the increase in Br. This effect increases as the baffle is nearer to the hot wall, that is the length-to-width ratio of passage 2 becomes large. From the figure it is also clear that at certain values of Br, Nu_h is zero and for greater values of Br, it becomes negative. This change in the sign of Nu_h is due to the change in the direction

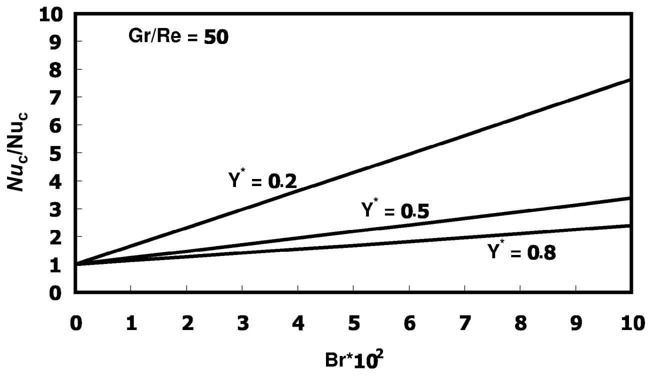

Fig. 4. Variations of Nu_c/Nu_c with *Br* (asymmetric heating).

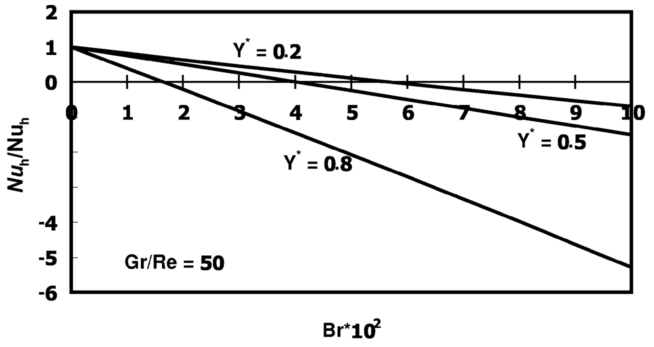

Fig. 5. Variations of Nu_h/Nu_h with *Br* (asymmetric heating).

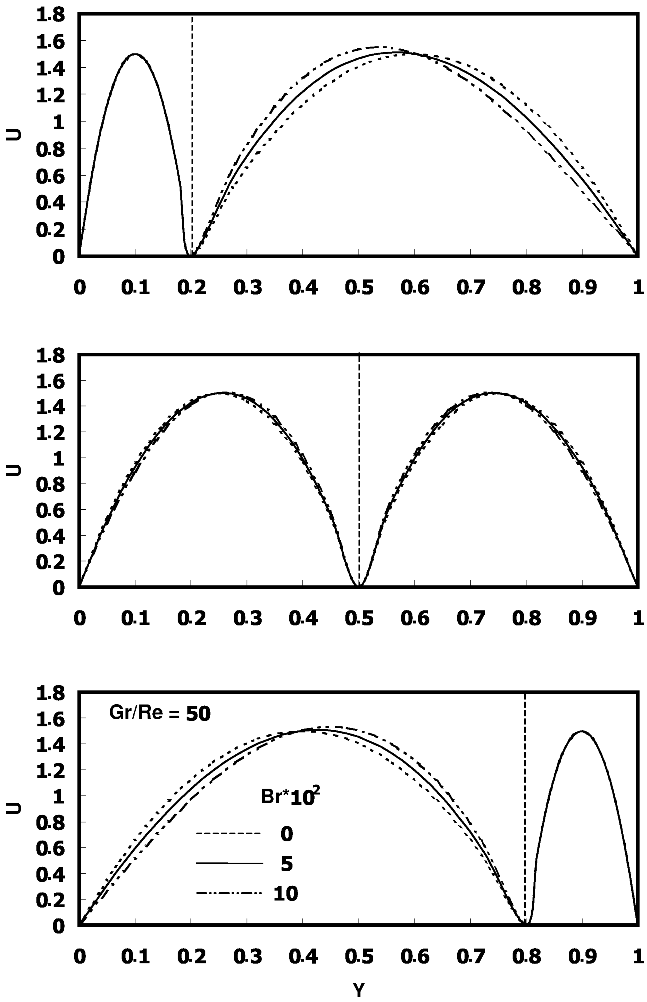

Fig. 6. Velocity profiles at different positions of the baffle (symmetric heating).

of heat flow at the hot wall when viscous dissipation is sufficiently large to make the temperature of stream 2 greater than the wall temperature T_h.

4.2. Symmetric heating

For symmetric heating, i.e., $T_c = T_h$, the effect of *Br* on the velocity profiles is similar to that for asymmetric heating. This can be observed in Fig. 6 which illustrates the velocity profiles at different positions of the baffle.

When viscous dissipation is negligible, i.e., $Br = 0$, the dimensionless temperature θ equals zero within the whole channel. For $Br > 0$, the dimensionless temperature increases significantly as can be seen in Fig. 7. The highest increase occurs around the baffle. The baffle represents an axis of symmetry when it is in the middle of the channel. Fig. 7 reveals that the temperature gradients increase in opposite directions at the channel walls. As a consequence, the Nusselt numbers also increase in opposite directions. Fig. 8 shows that Nu_c increases linearly in the positive direction as *Br* increases, while Fig. 9 shows that Nu_h increases linearly in the negative direction. The effect of the baffle position is similar to that for asymmetric heating. It may be noted that for symmetric heating, heat transfer is due to the energy generated by viscous dissipation only. When $Br = 0$, no heat is transferred in the channel.

5. Conclusion

The effect of viscous dissipation on fully developed laminar mixed convection in a vertical double-passage channel has been studied numerically. The channel is divided into two passages by means of a perfectly conductive, thin plane baffle and has uniform wall temperatures. Two cases are taken into consideration; asymmetric heating with different uniform wall temperatures and symmetric heating with equal uniform wall temperatures. An analytical solution has been developed for the case when viscous dissipation is negligible and the conditions for occurring a flow reversal has been deduced. The numerical solution gives very close values to those calculated using the analytical solution when viscous dissipation is neglected. The velocity and temperature profiles have been presented and the Nusselt numbers have been evaluated for both asymmetric and symmetric heating at different positions of the baffle.

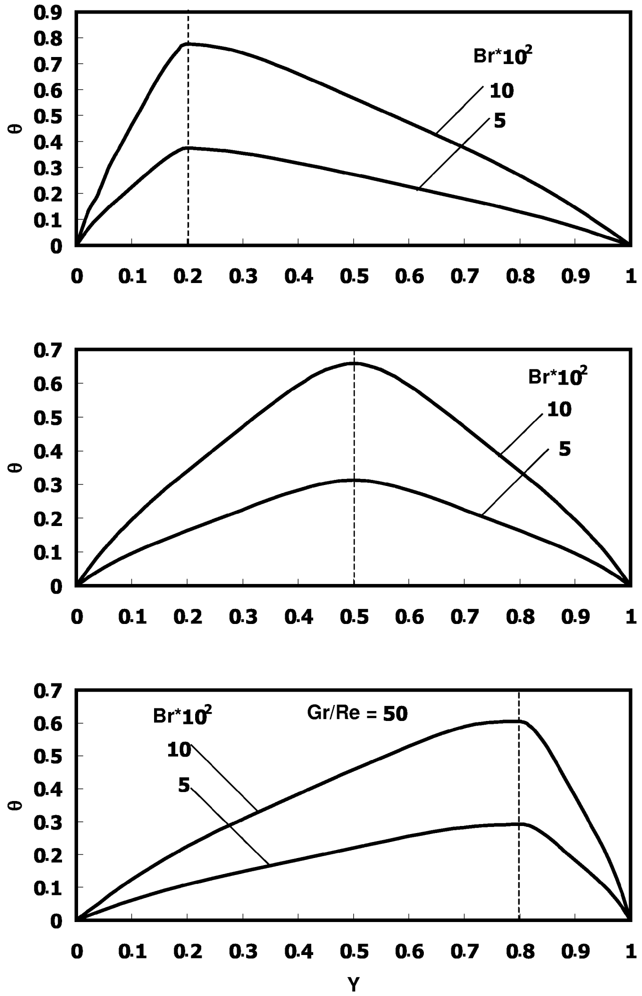

Fig. 7. Temperature profiles at different positions of the baffle (symmetric heating).

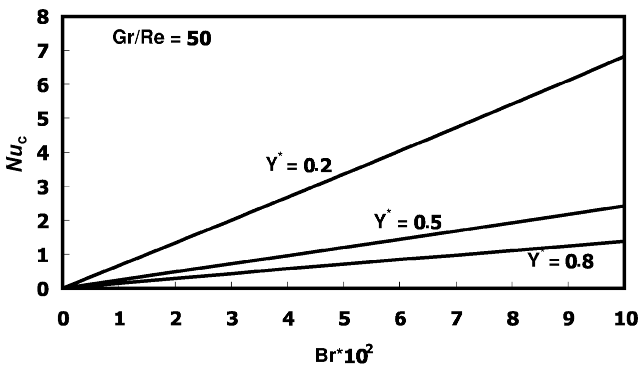

Fig. 8. Variations of Nu_c with Br (symmetric heating).

The results showed that:

(1) Brinkman number has a significant effect on the velocity profiles in the wider passage. In the narrower passage, its effect is negligible.
(2) When viscous dissipation cannot be neglected the temperature profile becomes nonlinear and depends on the baffle position.

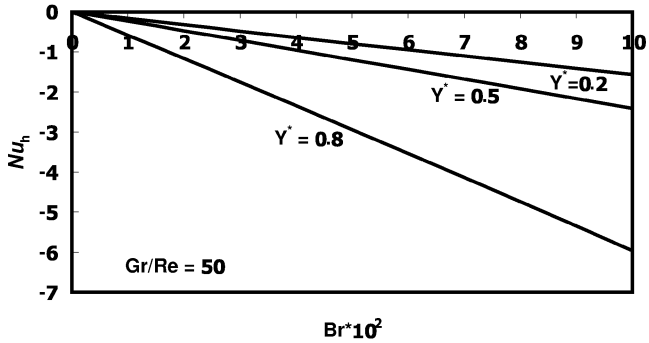

Fig. 9. Variations of Nu_h with Br (symmetric heating).

(3) The Nusselt number on the cool wall increases linearly with the increase in Brinkman number.
(4) The Nusselt number on the hot wall increases linearly with the decrease in Brinkman number. For asymmetric heating, Nu_h is positive at small values of Br. At certain values of Br, Nu_h is zero and for greater values of Br it becomes negative. For symmetric heating, Nu_h increases in the negative direction for all values of Br.
(5) Viscous dissipation enhances the chance of occurrence of flow reversal in passage 1, while it lowers this chance in passage 2.

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